

Problem 265

by Benjamin Bloch, Ph.D.

These Points Are Critical

When you're surveying across a "smooth" mountaintop, its *positive* slope becomes 0 at the top and then *negative* on the other side. The "change in the

slope" across the mountaintop is **decreasing**.

When you're surveying across a "smooth" valley, its *negative* slope becomes 0 at the bottom and then *positive* on the other side. The "change in the slope" across the bottom of the valley is **increasing**.

Using DD1 ("down decrease by one" is introduced in problem 257) to calculate dy/dx [also written as $y'(x)$] and equating it to 0 locates these peaks and valleys. Around a zero slope, the graph can rise and then fall, fall and then rise, or, after a pause, can continue in the same direction, called an *inflection point*. An example of an inflection point is a mountain that slopes down, levels off, and then continues its downward slope.

The x values where the slopes are zero are called "critical points."

Calculating the "change in the slope" (meaning applying a **second** DD1) can be used to determine whether a graph has maxima, minima, or inflection points.

Procedure: Calculate the slope $y'(x)$, by performing a DD1. Then, calculate its *changing* slope, $y''(x)$, by performing a second DD1.

To sum up:

- The first DD1 (first derivative) locates the instantaneous slope at every point on the curve.
- Equating the first derivative to 0 solves for the critical points, the x points on the curve where the slopes are 0.
- Performing another DD1 (second derivative) determines whether the 0 slopes are decreasing (a relative maximum), or increasing (a relative minimum), or are points of inflection.
- Equate the second derivative to zero, then evaluate by inserting their respective critical points' x values. Positive y'' values indicate minima, negative y'' values indicate maxima, and a 0 value can indicate a point of inflection. (For points of inflection, either sketch a few points on either side or check the signs of the first derivative slopes on either side.)

Consider a landscape modeled by the equation $y = x^3 - 3x^2 + 4$

1. What is the equation to determine its instantaneous slopes?
2. What are the critical values, the x values where the slopes = 0?
3. What is the equation for $y''(x)$, the change in the slope?
Caution: Do not insert x values before performing the second DD1.
4. What are the coordinates of the maximum (y'' negative) and minimum (y'' positive)?
5. Graph the equation to illustrate these answers.