Solution

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1. $y + \Delta y = -6(x + \Delta x)$;
   $-6x + \Delta y = -6x - 6\Delta x$;
   $\Delta y = -6\Delta x$
   $\Delta y/\Delta x = -6$.

   DD1: From $y(x) = -6x^1$; we get $y'(x) = -6x^0 = -6$.

2. $y + \Delta y = -(x + \Delta x)^2 + 5(x + \Delta x) = -x^2 - 2x\Delta x - (\Delta x)^2 + 5x + 5\Delta x$
   Substituting for $y(x) = -x^2 + 5x$
   We get: $-x^2 + 5x + \Delta y = -x^2 - 2x\Delta x - (\Delta x)^2 + 5x + 5\Delta x$
   Simplifying we get: $\Delta y = -2x\Delta x - (\Delta x)^2 + 5\Delta x$
   Dividing by Δx, we get: $\Delta y/\Delta x = -2x - \Delta x + 5$
   Finally, taking the limit as $\Delta x \to 0$, $dy/dx \equiv y'(x) = \lim_{\Delta x \to 0} (\Delta y/\Delta x) = -2x + 5$.

   DD1: From $y(x) = -x^2 + 5x$, we get $y'(x) = -2x + 5$.

3. $y + \Delta y = 4/(x + \Delta x)$
   Multiplying both sides by $(x + \Delta x)$ we get: $(y + \Delta y)(x + \Delta x) = 4$
   Expanding gives us: $yx + y\Delta x + x\Delta y + \Delta y\Delta x = 4$
   Since $yx = 4$, we get $y\Delta x + x\Delta y + \Delta y\Delta x = 0$
   Dividing by Δx; $y + x\Delta y/\Delta x + \Delta y = 0$
   Now, since the first and third terms $y + \Delta y = 4/(x + \Delta x)$, substituting, we get
   $4/(x + \Delta x) + x\Delta y/\Delta x = 0$
   Rearranging: $\Delta y/\Delta x = -4/x(x + \Delta x) = -4/(x^2 + x\Delta x)$
   Finally, taking the limit as $\Delta x \to 0$, $dy/dx \equiv y'(x) = \lim_{\Delta x \to 0} (\Delta y/\Delta x) = -4/x^2$.

   DD1: $y(x) = 4/x = 4x^{-1}$ thus $y'(x) = -4x^{-2} = -4/x^2$.