

problem corner

Problem 263

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Let's look at what "down decrease by one, DD1" instantly accomplishes. It is based on calculating the slope of a curve at a point using the "delta process."

The delta, Δ , process works as follows: For any function, say $y(x) = 3x^2$, if x changes by Δx , then we are interested in how y is effected by that change. In other words, wherever we see an x we add Δx , and wherever we see y we add Δy . Our function then becomes:

$$y + \Delta y = 3(x + \Delta x)^2$$

$$y + \Delta y = 3(x^2 + 2x\Delta x + (\Delta x)^2) = 3x^2 + 6x\Delta x + 3(\Delta x)^2$$

Now from the original function we know that $y(x) = 3x^2$, so we can replace the first y by $3x^2$. This yields: $3x^2 + \Delta y = 3x^2 + 6x\Delta x + 3(\Delta x)^2$

The $3x^2$ terms cancel and we are left with:
 $\Delta y = 6x\Delta x + 3(\Delta x)^2$

Dividing both sides by Δx , gives us $\Delta y/\Delta x = 6x + 3\Delta x$. Here is where the power of calculus comes in. We are interested in the slope at a point, the instantaneous slope, which means that we want to zoom in on the curve, or take the limit as Δx approaches 0. In that limit, that Δx approaches 0, we are left with:
 $\Delta y/\Delta x = 6x + 3\Delta x \rightarrow 6x$.

This is exactly the definition of the instantaneous slope: $dy/dx \equiv y'(x) = \lim_{\Delta x \rightarrow 0} (\Delta y/\Delta x)$.

DD1: From $y(x) = 3x^2$ then $y'(x) = 6x$.

The delta process provides the understanding while "down decrease by one, DD1" provides the "instantaneous" machinery.

Calculate the slope at each point of the following. Use the delta process, and check with down decrease by one, DD1.

1. $y(x) = -6x$
2. $y(x) = -x^2 + 5x$
3. $y(x) = 4/x$