Solution
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Let the 310 sagitta be "a" and the 120 sagitta be "b". Be = R–a, Bd = R–b and de = R.

$(R-b)^2+(R-a)^2=R^2$

$R^2-2Rb+b^2+R^2-2Ra+a^2=R^2$, or $R^2-2R(a+b)+a^2+b^2=0$

$R=\dfrac{2(a+b)\pm\sqrt{[2(a+b)]^2-4(a^2+b^2)}}{2}$, and for a=310 and b=120,

$R=\dfrac{2(310+120)\pm\sqrt{[2(310+120)]^2-4(310^2+120^2)}}{2}$, from which R= 702.764

Next, draw m parallel with AB through the leftmost small circle radius point and n parallel with BC through the rightmost small circle radius point.

$\dfrac{4r}{R}=\dfrac{m}{R-b}=\dfrac{n}{R-a}$; $\cos BAC=\dfrac{m}{4r}=\dfrac{4r(R-b)}{4rR}=\dfrac{(R-b)}{R}$, $\cos BCA=\dfrac{n}{4r}=\dfrac{4r(R-a)}{4rR}=\dfrac{(R-a)}{R}$

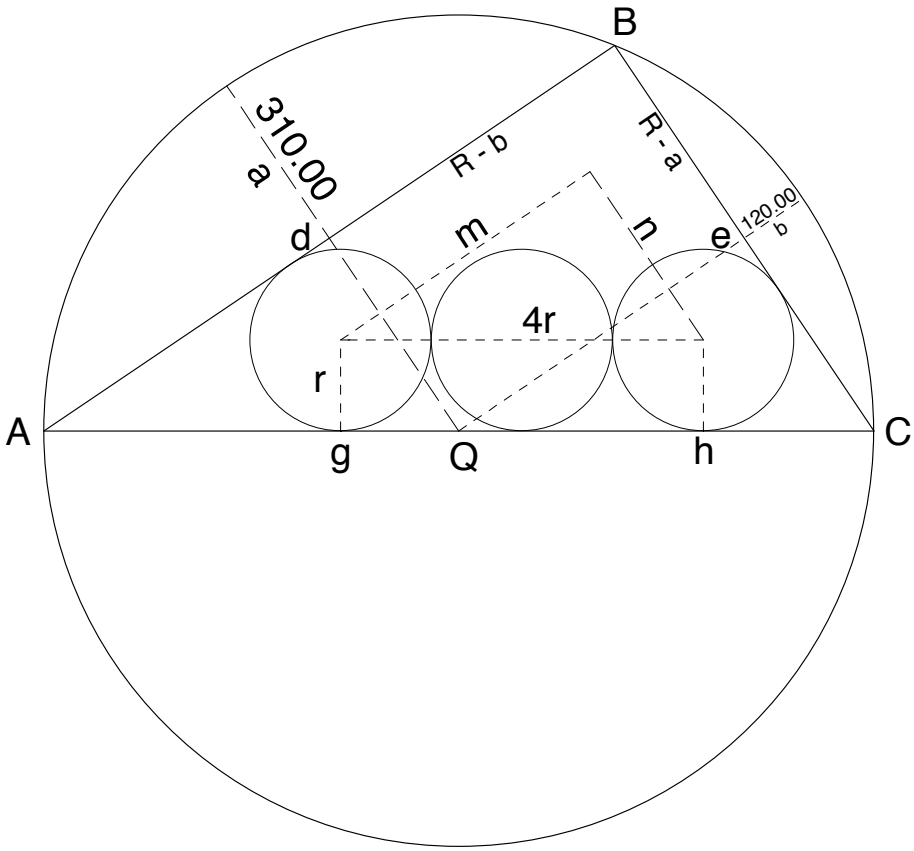
AC = Ag + gh + hC = $r\cdot\cot\dfrac{BAC}{2}+4r+r\cdot\cot\dfrac{BCA}{2}=2R$

$r\cdot\sqrt{\dfrac{1+\cos BAC}{1-\cos BAC}}+4\cdot r+r\cdot\sqrt{\dfrac{1+\cos BCA}{1-\cos BCA}}$ =2R = $r\cdot\sqrt{\dfrac{1+\dfrac{R-b}{R}}{1-\dfrac{R-b}{R}}}+4\cdot r+r\cdot\sqrt{\dfrac{1+\dfrac{R-a}{R}}{1-\dfrac{R-a}{R}}}$

For the known values,

$$r\cdot\sqrt{\frac{1+\dfrac{702.764-120}{702.764}}{1-\dfrac{702.764-120}{702.764}}}+4\cdot r+r\cdot\sqrt{\frac{1+\dfrac{702.764-310}{702.764}}{1-\dfrac{702.764-310}{702.764}}}=2\cdot 702.764$$

$$r\cdot\sqrt{\frac{1.82924566426}{0.17075433574}}+4r+r\cdot\sqrt{\frac{1.558884633268}{0.44111536732}}=1405.528$$

3.27303·r + 4·r + 1.87988·r = 1405.528, and r=153.561

AC=2R=1405.528; BC=2Be=2(702.764–310) =785.528; AB =2Bd= 2(702.764–120)=1165.528