



# The Earth Is Round (But not in Kansas) ... or Down Decrease by 1 (DD1)

Calculus can be a slippery slope, so let's see how this slide can be made easy and fun. First, some terminology.

Consider two variables,  $x$  and  $y$ , where  $x$  is the independent variable and  $y(x)$  is the dependent variable. Writing  $y$  as  $y(x)$  clearly indicates that the value of  $y$  depends upon the value of  $x$ ; in other words,  $y$  is a function of  $x$ .

We will start with the general form:  $y(x) = ax^n$  where " $a$ " and " $n$ " are numbers.

Example: if  $a = 1.5$  and  $n = 2$ , then  $y(x) = 1.5x^2$ .

We will require symbols that differentiate between relatively large and small changes in the variables. (The meaning of the word "relatively" will be defined later.)

The symbol  $\equiv$  means "defined as."

The symbol  $\Delta$  (delta) means "change" so that  $\Delta y$  (delta  $y$ ) means "the change in  $y$ ," or  $(y_2 - y_1)$ , the difference between two values of  $y$ .

$\Delta x$  (delta  $x$ )  $\equiv$  "the change in  $x$ " or  $(x_2 - x_1)$ , the difference between two values of  $x$ .

$\Delta y/\Delta x$  = the change in  $y$  divided by the change in  $x$ , called the slope, where  $y$  (ordinate) is the height axis and  $x$  (abscissa) is the width axis.

$dx$   $\equiv$  a minuscule change in  $x$ , (imagine that  $x$  represents your present salary and that  $dx$  represents your raise.)  $dy$   $\equiv$  a minuscule change in  $y$ .

Finally,  $dy/dx$  (written as  $y'(x)$ ) is the value of the slope of the tangent line at point " $x$ " on the curve. It is called the instantaneous slope.

We can easily calculate the instantaneous slope, at any point  $x$ , for all functions of the form  $y(x) = ax^n$  where " $a$ " and " $n$ " are numbers, instantly!

In other words, to calculate  $y'(x) \equiv dy/dx$  for any point  $x$  on the graph of  $y(x) = ax^n$  for any " $a$ " and any " $n$ ."

Here is the mantra: DOWN DECREASE BY ONE (DD1)

Here is how it works: To calculate  $y'(x)$  from  $y(x) = ax^n$ , bring the  $n$  down as a multiplier and reduce the exponent value by 1, thus  $y'(x) = nax^{n-1}$

That is all there is to it!

In the example,  $y(x) = 1.5x^2$ ,  $a = 1.5$  and  $n = 2$ .

Down decrease by one means:  $y'(x) = 2(1.5)x^{2-1} = 3x$ . Again, the slope of the curve  $y(x) = 1.5x^2$  at any point  $x$  is:  $y'(x) = 3x$ .

At the point  $x = 1$ ,  $y'(1) = 3$ , at  $x = 2$ ,  $y'(2) = 6$ , and so forth.

*Congratulations!* You have just completed the basic operation of differential calculus.

Let's take a closer look by looking at the graph of example  $y(x) = 1.5x^2$ . We illustrate this graph without units.

To the right is a graph of a possible ski slope,  $y = 1.5x^2$  where the  $x$ -axis scale is from 0 to 40 and the  $y$ -axis scale is from 0 to 2,250. We will anchor the point  $P(x=15, y=337.5)$  on the graph so that it does not move.

## Problem:

Determine the  $x$  and  $y$  coordinates of the point on the graph  $y(x) = 1.5x^2$  where the instantaneous slope equals:

1. 75
2. 60

Calculate the areas under the straight line connecting points:

3.  $P$  and  $P_2$
4.  $P$  and  $P_1$

Our task is to calculate the value of the slope exactly at point  $P$ . Because slope is defined as height divided by run, or the difference in the  $y$  distance divided by the difference in the  $x$  distance, we have a problem because the ski slope at point  $P$  is not a straight line. To approximate the slope at  $P$ , let's imagine a fictitious skier whose back end of his skis is anchored at point  $P$  and whose front end extends to point  $P_2(x_2=35, y_2=1,837.5)$ . As can be seen from the graph, the slope of the green line skis does not well match the variation in the slope on the actual curve.

The slope of the green line is given by:

$$\Delta y/\Delta x = (y_2 - y)/(x_2 - x) = (1,837.5 - 337.5)/(35 - 15) = 1500/20 = 75.$$

Now, suppose we choose shorter skis so that the front end of our skis reach point  $P_1(x_1=25, y_1=937.5)$ . Notice now that this red line of our shortened skis is a closer match to the actual ski slope, closer, but not exactly.

The slope of the red line is given by:

$$\Delta y/\Delta x = (y_1 - y)/(x_1 - x) = (937.5 - 337.5)/(25 - 15) = 600/10 = 60.$$

Now, like moving from the Earth to Kansas, use shorter and shorter skis until finally the slope is a straight line. The skis need not be made any shorter from this point on, because it will remain a straight line with constant slope. We have succeeded in getting the slope at any point on that straight line. Whatever the curve, zoom in so close that the curve becomes a straight line.

Now let us use calculus instead to determine the value of the slope at point  $P(15, 337.5)$  for the ski graph  $y(x) = 1.5x^2$ . "Down decrease by one" gives us  $y'(x) = 2(1.5)x = 3x$ . We are *finished*. So that  $y'(15) = 3(15) = 45$ .

Thus, using  $y'(x) = 3x$  allows the calculation of the instantaneous slope at any point  $x$  on the graph of  $y(x) = 1.5x^2$ .

