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by David Lindell, LS

In triangle ADQ, DQ is the centerline radius, R, and AQ is the centerline radius plus 50'.
The bearing of AQ = N 42°00'47" W (it's the BC radial and perpendicular to the tangent).

$R^2 = 496.28^2 + (R+50)^2 - 2 \cdot 496.28 \cdot (R+50) \cdot \cos 43°42'18"$

$R^2 = 246293.84 + R^2 + 100R + 2500 - 717.528R - 35876.42$, from which R=344.790'

In triangle ADQ, by the Law of Sines: $\frac{\sin 43°42'18"}{344.790} = \frac{\sin AQD}{496.28} = \frac{\sin ADQ}{394.790}$

so that angle AQD = 84°00'09" and angle ADQ = 52°17'33"

Angle CDE = 85°43'05" + 11°45'38" = 97°28'43"
Angle QDE = 52°17'33" + 97°28'43" = 149°46'16"

In triangle QED, also by the Law of Sines: $\frac{\sin 149°46'16"}{394.790} = \frac{\sin EQD}{ED} = \frac{\sin QED}{344.790}$

so that angle QED = 26°05'04", angle EQD = 4°08'40" and ED = 56.67'

In triangle QCD, $\frac{\sin 52°17'33"}{294.790} = \frac{\sin QCD}{344.79} = \frac{\sin CQD}{CD}$

Angle QCD = 67°43'06" and/or 112°16'54", the latter being used for this problem.
Angle CQD is then 15°25'33" and side CD is 99.11'

In triangle ABQ, $\frac{\sin 43°42'18"}{294.790} = \frac{\sin ABQ}{394.79} = \frac{\sin AQB}{AB}$ and angle AQB = 24°00'48"

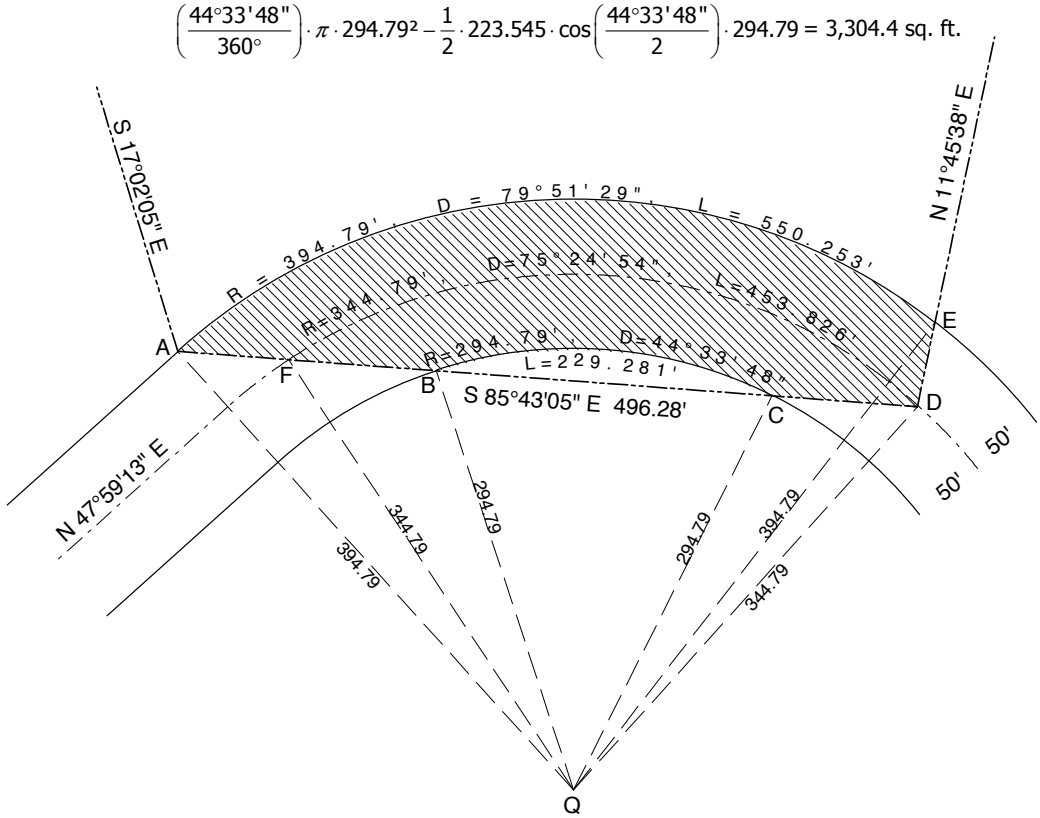
And side AB = 173.62'
(Note that angle ABQ=112°16'54" as it should be because triangle QBC is isosceles)
In triangle QBC, BC is the chord of a circle of radius 294.79' with a central angle of 180° – 2x67°43'06"= 44°33'48", or 223.55'.
AB + BC + CD = 173.62' + 223.55' + 99.11' = 496.28'

In triangle QFB, $\frac{\sin 112°16'54"}{344.790} = \frac{\sin BFQ}{294.79} = \frac{\sin BQF}{BF}$ so angle BFQ = 52°17'33"

and angle BQF = 15°25'33" as a check and side FB = 99.11'
Arc AE with a radius of 394.79' and Δ = 79°51'29" has an arc length of 550.253'
Arc FD with a radius of 344.79' and Δ = 75°24'54" has an arc length of 453.826'
And arc BC with a radius of 294.79' and Δ = 44°33'48" has an arc length of 229.281'

The area of AEDF is therefore $\frac{550.253 + 453.826}{2} \cdot 50 = 25{,}102.0$ sq. ft. and the area of

FDCB is $\frac{453.826 + 229.826}{2} \cdot 50 = 17{,}077.7$ sq. ft. for a total of 42,179.7 sq. ft.

The area between the radius of 294.79' and the property line is

$\left(\frac{44°33'48"}{360°}\right) \cdot \pi \cdot 294.79^2 - \frac{1}{2} \cdot 223.545 \cdot \cos\left(\frac{44°33'48"}{2}\right) \cdot 294.79 = 3{,}304.4$ sq. ft.