problem corner solution

Solution to Problem **158**

by Dave Lindell, L.S.

1) $J^2 - 2 = H$

Let $J = 2 \cos m$

$2^2 \cos^2 m - 2 = H$

$2(2 \cos^2 m - 1) = H = 2 \cos 2m$

(In general, $\cos 2a = 2 \cos^2 a - 1$)

2) $H^2 - 2 = -B$

$(2\cos 2m)^2 - 2 = -B$

$2(2\cos^2 2m - 1) = -B$

$2 \cos 4m = -B$

3) $B^2 - 2 = -T$

$(-2)^2\cos^2 4m - 2 = -T$

$2(2 \cos^2 4m - 1) = -T$

$2 \cos 8m = -T$

4) $T^2 - 2 = J$

$4\cos^2 8m - 2 = J$

$2(2\cos^2 8m - 1) = 2 \cos m$

$\cos 16m = \cos m$

In general, $\cos (s + t) = (\cos s)(\cos t) - (\sin s)(\sin t)$

and $\cos (s - t) = (\cos s)(\cos t) + (\sin s)(\sin t)$

$\cos (s + t) - \cos (s - t) = -2(\sin s)(\sin t)$

Let $s + t = x$ and $s - t = y$, so that $s = ½ (x + y)$, $t = ½ (x - y)$

$\cos x - \cos y = -2 \sin [½ (x + y)] \sin [½ (x - y)]$

if $\cos 16m = \cos m$, $\cos 16m - \cos m = 0$

$$\cos 16m - \cos m = -2 \sin \frac{16m+m}{2} \sin \frac{16m-m}{2} = 0$$

$-\sin [(16m + m)/2] = 0$ and $\sin[(16m - m)/2] = 0$

$\sin 8.5m = 0$ and $\sin 7.5m = 0$

$\sin m$ is 0 when $m = 0, 180, 360, 540, 720, 900$ etc.

m is then 0° or 21°10′35.3″ or 24° or multiples thereof.

$\cos m$ and $\cos 2m$ must be positive and $\cos 4m$ and $\cos 8m$ must be negative, satisfied only if $m = 24°$

$J = 2 \cos 24° = 1.827090915$, $T = 1.9562955201$,

$B = 0.209056927$, $H = 1.338261213$

Solution to Problem

159

by Benjamin Bloch, Ph.D.

Single Digit Quality: Factorials

The factorial of a number is the value of that number when it is multiplied by all of its preceding numbers. The factorial operation is designated by an exclamation mark. Thus, 5 factorial means the value obtained by the operation 5 x 4 x 3 x 2 x 1 and is written as 5!

The following table lists the first 22 numbers and their factorial values. Your problem is to fill in the missing values for the SUM OF DIGITS and SDQ columns.

Review: SDQ or Single Digit Quality for any number is obtained by adding all of the digits until a single digit remains, the SDQ of that number.

SDQ OF FACTORIALS

n	n!	SUM OF DIGITS	SDQ
1	1	1	1
2	2	2	2
3	6	6	6
4	24	6	6
5	120	3	3
6	720	*9*	*9*
7	5,040	*9*	*9*
8	40,320	*9*	*9*
9	362,880	*27*	*9*
10	3,628,800	*27*	*9*
11	39,916,800	*36*	*9*
12	479,001,600	*27*	*9*
13	6,227,020,800	*27*	*9*
14	87,178,291,200	*45*	*9*
15	1,307,674,368,000	*45*	*9*
16	20,922,789,888,000	*63*	*9*
17	355,687,428,096,000	*63*	*9*
18	6,402,373,705,728,000	*54*	*9*
19	121,645,100,408,832,000	*45*	*9*
20	2,432,902,008,176,640,000	*54*	*9*
21	51,090,942,171,709,440,000	*63*	*9*
22	1,124,000,727,777,607,680,000	*72*	*9*